Statistics I (Eco 255)

Formula Sheet for Second Exam

1. Definitions:

   \( S = \) The universal set. All possible outcomes of the experiment are elements of the universal set.

   \( \Phi = \) The empty set. A set that has no points.

   \( e_i = \) Simple events which denote all the possible outcomes of the experiment. \( \sum e_i = S \) and \( e_i \cap e_j = \Phi \).

   \( \cup = \) The union of two events, \( e_i \cup e_j \) indicates the set which has occurred if \( e_i \) or \( e_j \) (or possibly both) have occurred.

   \( A = \) A compound event defined as the union of simple events. We say the event \( A \) has occurred if any other simple events which comprise \( A \) has occurred.

   \( \Pr\{e_i\} = \) A probability distribution defined over the set of basic outcomes. \( 0 \leq \Pr\{e_i\} \leq 1 \).

2. Probability

2.1 Basic Formulae

   \( P(A) = \sum_{e_i \in A} \Pr\{e_i\} \) \quad \( \Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B) \)

2.2 Compliments

   \( \Pr(A^c) = 1 - \Pr(A) \) \quad \( \Pr(A_1 \cup A_2 \cup \ldots \cup A_n) = 1 - \Pr(A_1^c \cap A_2^c \cap \ldots \cap A_n^c) \)

2.3 Conditional Probability and Independence

   \( \Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)} \) \quad \( \Pr(A \cap B) = \Pr(A|B) \Pr(B) = \Pr(B|A) \Pr(A) \)

Sets \( A \) and \( B \) are independent if and only if

   \( \Pr(A \cap B) = \Pr(A) \Pr(B) \)

2.4 Permutations and Combinations

   \( n! = n \cdot (n - 1) \cdot (n - 2) \cdots 2 \cdot 1 \) \quad \( P_r^n = \frac{n!}{(n-r)!} \) \quad \( C_r^n = \frac{n!}{r!(n-r)!} = \binom{n}{r} \)

2.5 Bayes Formula

   \( \Pr(A_1 | B) = \frac{\Pr(B | A_1) \Pr(A_1)}{\Pr(B | A_1) \Pr(A_1) + \Pr(B | A_2) \Pr(A_2) + \ldots + \Pr(B | A_n) \Pr(A_n)} \)
2.6 Expectations

\[ E(x) = \sum_{x} x \Pr(x) = \mu \quad E(x - \mu)^2 = \sum_{x} (x - \mu)^2 \Pr(x) = \sigma^2 \]

3. Discrete Distributions

3.1 Binomial Distribution

Let \( x \) denote the sum of the results from \( n \) Bernoulli trials. The probability of \( x \) successes in \( n \) trials, where \( p \) is the probability of a ‘success’ and \( q = 1 - p \), has the following form:

\[ x = \sum_{j=1}^{n} x_j \quad \Pr(x) = p^x q^{n-x} \quad E(x) = np \quad \sigma^2 = npq \]

3.2 Poisson Distribution

Let \( x \) denote the number of ‘successes’ or ‘arrivals’ in some interval of time or space. The probability of that \( x = k \) has the following form.

\[ \Pr(x) = \frac{\mu^k e^{-\mu}}{k!} \quad E(x) = \mu \quad \sigma^2 = \mu \]

3.3 Summary

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Parameters</th>
<th>Trials</th>
<th>Range</th>
<th>Mean</th>
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<tbody>
<tr>
<td>Binomial</td>
<td>( n, p )</td>
<td>( n )</td>
<td>( x = 0, 1, \ldots, n )</td>
<td>( np )</td>
<td>( npq )</td>
</tr>
<tr>
<td>Poisson</td>
<td>( \mu )</td>
<td>?</td>
<td>( x = 0, 1, \ldots, n )</td>
<td>( \mu )</td>
<td>( \mu )</td>
</tr>
</tbody>
</table>

4. Continuous Distributions:

4.1 The Uniform Distribution:

A random variable is said to be uniformly distributed between \( a \) and \( b \) if

\[ f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{elsewhere} \end{cases} \]

The mean of this distribution is \((b + a)/2\), and the variance is \((b - a)^2/12\).

4.2 The Normal Distribution:

The normal distribution is our bell-shaped curve. Its exact form depends upon two parameters, \( \mu \) and \( \sigma \). Two useful formula relate the standardized and nonstandardized forms:

\[ z = \frac{x - \mu}{\sigma} \quad x = z\sigma + \mu \]