Solutions to Binomial Valuation Problems

1. This problem demonstrates the method for finding state prices, and their relation to traditional risk-adjusted rates.

   (a) The state prices are embedded in the two equations:

   \[ \begin{align*}
   40 &= 60Q_u + 24Q_d \\
   1 &= 1.1Q_u + 1.1Q_d
   \end{align*} \]

   The best (least inaccurate) solution procedure is to multiply the second equation by \((24 / 1.1) = 21.81818182\). This leads to:

   \[ \begin{align*}
   40 &= 60Q_u + 24Q_d \\
   21.8182 &= 24Q_u + 24Q_d
   \end{align*} \]

   with solutions, \(Q_u = .505, \quad Q_d = .404\).

   (b) The expected cash flow of the security is 60(.6) + 24(.4) = $45.60 and the expected return is +14%.

   (c) The future price would be $44 (because this is a financial asset). This can be established by setting initial value of the future cash flows equal to zero. In particular,

   \[ 0 = .505 \times (60 - F) + .404 \times (24 - F) = 40 - .909F \]

2. This problem illustrates the simplicity of the state pricing approach as applied to options valuation.

   (a) The payoffs to the call are $20 (up state) and zero (down state). The value of the call at time zero is

   \[ C_0 = 20 \times (.505) + 0 \times (.404) \]

   = $10.10

   (b) The rate of return is +90.1% (up) and −100% (down).

   (c) The payoffs to the put are $0 (up state) and $16 (down state). The time zero value of the put is

   \[ P_0 = 0 \times (.505) + 16 \times (.404) \]

   = $6.46

   (d) The rate of return is −100% (up) and +147.5% (down).

   (e) Yes:

   \[ 40 + 6.46 - 10.10 = \frac{40}{1.1} = 36.36 \]

3. The $60 share price at time one branches to $90 and $36 at time two. The $24 time one price branches to $36 and $14.40 at time two. The solutions to this problem are:

   (a) \( C_0 = 12.75 \quad (C_u = 25.25 \quad C_d = 0.00) \).
(b) $C_0 = 12.75$ (no value to premature exercise.)

(c) $P_0 = 5.81$ \hspace{1cm} ($P_u = 1.62$ \hspace{1cm} $P_d = 12.36$). Note that the two period american put is less valuable than the one period put of the previous problem.

(d) $P_0 = 7.28$ \hspace{1cm} ($P_u = 1.62$ \hspace{1cm} $P_d = 16.00$ premature exercise).

(e) No! See the calculation below. The concept of put call parity requires that the options be held to expiration. This is normally violated when an american put is involved.

(f) Yes!

$$40 + 5.81 - 12.75 = \frac{40}{(1.1)^2} = 33.06$$

4. This problem demonstrates one of the two approaches to incorporating dividends into the model. The other approach is to assume a constant dividend yield (See section 17.3). The diagram for the stock will drop from 60 to 56, ex-dividend at time one, and then branch to a final result of $84 or $33.60. The lower paths have the stock dropping to $20 ex-dividend, then branching to $30 or $12. Note that the overall effect of the dividend of the dividend is to depress the final stock price by disinvesting you (of course, you can correct this by reinvesting the dividends but the option owner can’t).

(a) $C_0 = 11.22$ \hspace{1cm} ($C_u = 22.22$ \hspace{1cm} $C_d = 0.00$).

(b) $C_0 = 11.22$ (the dividend isn’t large enough to cause exercise).

(c) $P_0 = 7.92$ \hspace{1cm} ($P_u = 2.59$ \hspace{1cm} $P_d = 16.36$). Note that the two period american put has increased in value from the previous problem because of the dividend.

(d) $P_0 = 9.39$ \hspace{1cm} ($P_u = 2.59$ \hspace{1cm} $P_d = 20.00$ premature exercise after the dividend).

(e) No! See the calculation below. The concept of put call parity requires that the options be held to expiration. This is normally violated when an american put is involved.

(f) Yes — corrected for dividends — with a .01 rounding error!

$$40 + 7.92 - 11.22 = \frac{40}{(1.1)^2} + \frac{4}{1.1} = 36.69$$